
STUDIES

Article received on March 5th 2024

Ale accepted on June 28th 2024

Original scientific paper

UDC 115:7.01

519.246.8

DOI: 10.5937/newso2463017M

Miloš Milovanović*

Mathematical Institute of the Serbian Academy of Sciences and Arts

Irena Kuletin Čulafić**

University of Belgrade, Faculty of Architecture

Department for History and Theory of Architecture and Art

Nicoletta Saulig***

Juraj Dobrila University of Pula, Faculty of Engineering

VERTICAL TIME IN ARTS

Abstract: The paper sheds light on Gaston Bachelard's concept of vertical time. The topic is elaborated in relation to intrinsic time by Étienne Souriau which concerns the plastic arts. It corresponds to the time operator formalism of complex systems whose defining feature is the existence of the intrinsic time operator. The complex system model is based upon statistical causality which occurs through inheritance in the vertical temporality. The model is derived for one-dimensional signals, but it is

* The author's contact details: milosm@mi.sanu.ac.rs.

*** The author's contact details: kuletin@arh.bg.ac.rs.

*** The author's contact details: nicoletta.saulig@unipu.hr.

easily generalizable to two-dimensional and three-dimensional ones. The aesthetics of verticality is shown to be significant for music, poetry, architecture and film as well.

Keywords: creativity, fractal geometry, self-organization, time operator, statistical causality

Introduction

The concept of vertical time, which is an experience of temporality related to artistic creation, has been defined by the French philosopher Gaston Bachelard (1884–1962). In his work pioneering historical epistemology, reflection on time is a recurrent subject. Whilst Bachelard identifies a horizontal timeline as illustrated by clocks, social, phenomenal and vital frameworks of duration, to be prosodic, the time of poetry is a vertical one which surges up the instant in a metaphysical scope. In that respect, he wrote: “Every true poem can reveal the elements of suspended time, meterless time – a time we shall call vertical in order to distinguish it from everyday time, which sweeps along horizontally with the streaming waters and the blowing winds.”¹ A rupture of the timeline that spreads into the dormant depth of temporality is described as an ecstatic moment, in which past and future converge. Bachelard also pointed out the eternal capabilities of vertical time: “This line running perpendicular to the temporal axis of life alone, in fact, gives consciousness of the present, the means to flee and escape, to expand and deepen, which have very often led to the present instant being linked to an eternity.”²

According to Bachelard, an instant to be complex requires organization into vertical architecture corresponding to the creative process. Verticality is complex due to the ability to condense antithesis:

It is essentially a harmonic relation between two opposites. Within a poet's passionate instant, there is always a touch of reason; within the reasoned rejection, always a touch of passion. Successive antitheses already fill a poet with pleasure. But for these antitheses to yield an experience of rupture and ecstasy, they must contract into ambivalence. Only then does a poetic instant arise.³

Although Bachelard had earned a licence in mathematical science from the Lycée Saint-Louis in Paris, he did not propose a definition of vertical time in

¹ Gaston Bachelard, *Intuition of the Instant*, Evanston, Northwestern University Press, 2013, 58.

² Gaston Bachelard, *The Dialectic of Duration*, London, Rowmann& Littlefield International, 2016, 95.

³ Gaston Bachelard, 2013, op. cit., 59.

terms of mathematics. As a matter of fact, there are very few attempts to elucidate such a conception mathematically and one of them has been proposed by Saulig.⁴ The current article significantly extends previous consideration, in order to provide a complex system model of the vertical temporality. In that regard, it should become a universal concept corresponding to artistic creativity which is opposed to Kramer's view that verticality concerns particular practices of art only.⁵ Authors have also demonstrated a significant link to time's arrow which is a crucial problem in the physical science.⁶

The main reason for the lack of a model is the extreme complexity of the issue it refers to. As opposed to the horizontal timeline, Bachelard highlights the time of poetry spreading into depth and height, which is an extra dimension that implies the vertical structuring inherent to architecture. In his view, imagination inhabits a complex instant which is vertically designed.⁷ An experience of "rupture" and "ecstasy" should reorganize the horizontal timeline in a vertical temporality, evoking the mythological context of Jacob's ladder, which is a scale that connects Earth to Heaven.⁸ It corresponds to the temporal depth of the icon as elaborated by Antonova, who followed the research which had been initiated by Pavel Florensky a century before.⁹ Milovanović and Tomić have supplemented her elaboration in terms of fractal geometry and complex systems.¹⁰ An aesthetical criterion that unifies the traditional iconography and physics of complex systems has also been initiated by Milovanović and Medić-Simić.¹¹

⁴ Nicoletta Saulig, "Vertical Time: Sound and Vision", *Kragujevac Journal of Mathematics*, 47(7), 2023, 1065–1074.

⁵ Jonathan D. Kramer, *The Time of Music*, New York, Schirmer Books, 1988, 386–388.

⁶ Miloš Milovanović, Nicoletta Saulig, "The Duality of Psychological and Intrinsic Time in Artworks", *Mathematics*, 12(12), 2024, 1850.

⁷ Gaston Bachelard, *On Poetic Imagination and Reverie*, Putman, Spring Publications, 1971, 84–85.

⁸ Richard Kearney, "Vertical Time: Bachelard's Epiphanic Instant", in: Eileen Rizo-Patron (Ed.), *Adventures in Phenomenology: Gaston Bachelard*, New York, Sunny Press, 2017, 49–61.

⁹ Clemena Antonova, *Space, Time and Presence in the Icon*, Farnham, Ashgate Publishing Ltd., 2010.

¹⁰ Miloš Milovanović, Bojan M. Tomić, "Fractality and Self-organization in the Orthodox Iconography", *Complexity*, 21(S1), 2016, 55–68.

¹¹ Miloš Milovanović, Gordana Medić-Simić, "Aesthetical Criterion in Art and Science", *Neural Computing and Applications*, 2021, 33(6), 2137–2156.

The complex system model should be elaborated following on from Étienne Souriau in the consideration of the spatio-temporal structure.¹² His concept of intrinsic time presents an extension of Bachelard's conception regarding plastic arts, which has surpassed any separation into the arts of space and arts of time. But the link between the horizontal and vertical axis in spatio-temporal structuring has not been resolved either by Bachelard, or by Souriau who, moreover, claimed that a study of the musical space was yet to be done. In that regard, the investigation of music is an issue of particular significance and possibilities.¹³ Some attempts to consider the temporality of music have recently been made.¹⁴ However, the current article does not support such a constructivist viewpoint. It is built upon the original conception by Bachelard, Souriau and Florensky, which is not about any time making but the creation and observation of an artwork.

The introductory section concerned the research subject. Section 2 is aimed at establishing the horizontal and vertical axis of space-time in regard to music, which should shed light on the concept of intrinsic time. Section 3 presents the time operator formalism of complex systems offering a model of the artwork spatio-temporality. In Section 4, there is a comprehensive discussion concerning the interpretation of the model in terms of music, poetry, architecture and film as well. The last section offers the concluding remarks.

Materials and Methods

Music and Myth

In order to discuss the structure of the myth, Claude Lévi-Strauss referred to Ferdinand de Saussure, who had considered language via two complementary aspects – *langue* and *parole* – the first of which was ruled by reversible and the second, by irreversible time.¹⁵ According to his view, mythology is a system that integrated both in order to disclose the complex instant which demands statistical causality to be taken into account.¹⁶

¹² Étienne Souriau, "Time in the Plastic Arts", *Journal of Aesthetics and Art Criticism*, 7(4), 1949, 294–307.

¹³ Dragutin Gostuški, *Vreme umetnosti: prilog zasnivanju jedne opšte nauke o oblicima*, Beograd, Prosveta, 1968, 191.

¹⁴ Guerino Mazzola, Alex Lubet, Yan Peng, Jordan Goebel, Christopher Rochester, Sangeeta Day, *Making Musical Time*, Springer, 2021.

¹⁵ Claude Lévi-Strauss, "The Structural Study of Myth", *Journal of American Folklore*, 68(270), 1955, 428–444; Ferdinand de Saussure, *Course in General Linguistics*, New York, Philosophical Library, 1959, 98–100.

¹⁶ Miloš Milovanović, Gordana Medić-Simić, op. cit.

In the same manner, Massimo Donà regards music to be a primordial language expressing the substance of the myth.¹⁷ It pulsates in a proper vibration which is aligned to the dynamism of *physis* (in Greek, φύσις – origin, birth, nature). The mission of the musician is rather listening than playing since “sonority is an authentic image of life”. In that respect, one should realize the claim by Ludwig van Beethoven: “Life primordially corresponds to a sonorous vibration and the human to a string instrument.”¹⁸

In Donà's view, motion is a substantial feature of music which is labelled with the term *temporality*. Aesthetic sensation is possible for the sake of the time jolts it is interwoven with, which reflects the truth about one's proper and ever varying self. Just at the pace of music, the diversity is regarded to be an authentic state of identity which is significant for the aesthetic enjoyment. It is about time that is transient and partial, whose perfection is determined by the suspension and incompleteness of what is concretely present, and therefore by its ability to predict the future so well or remember the past. That's why “jazz creates both the concrete and the extraordinary but also the paradoxical experience, whilst this is not done by classical music” whose temporality “is inextricably dependent on the space whose primacy has never been challenged”.¹⁹

Such a viewpoint is based upon philosophical sources concerning the relationship between cosmology, arts and creativity. The Pythagoreans considered *heavenly music* the one initiated by the celestial spheres, whose sound has the perfect articulation of the dynamical *logos* (in Greek, λόγος – ratio, verb, truth, statute, measure) that releases matter from a static determination. According to Plato, music has gnoseological as well as aesthetic value, since priority is not given to ears over the mind. Such music is appropriate that generates full scale reality from itself and reality. Since drawing from the sacred domain of mythology, it succeeds in transcending the gap between unity and multiplicity, between identity and diversity, between goodness and beauty, recalling Bachelard's definition of the poetic instant, into which all opposing, complementary and fraternal forces flow.

Identification of a primordial *aporia* (in Greek, ἀπορία – impass, puzzlement), that concerns the one and the multitude pointing its transcendence, is reported by Plato in the *Parmenides*. Transcending the *aporia*, music is able

¹⁷ Massimo Donà, *Filosofiadellamusicca*, Milano, Tascabili Bompiani, 2006, 10–12.

¹⁸ Ludwig van Beethoven, *Autobiografia di un genio – Lettere, pensieri, diari*, Milano, Mondadori, 2005, 115.

¹⁹ Massimo Donà, op. cit., 24–26.

to perform aesthetic *dianoia* (in Greek, δίανοια – ratio, significance) which means a formative structure that is vertically designed. It is perceived by an intellect *in vivo*, always and only due to the musical *magic* (in Greek, μαγικός–wonder). In that respect, Plato advises musicians that every play and song should be proclaimed sacred.²⁰

Saint Augustine published a significant treatise *On Music*, which he considered Christian art in the true sense. In his view, music is the science of proper measurement designed by a free rhythm. What one likes in rhythmic sensation is metrics measured by equality.²¹ Augustine's creed is designed upon the concept of time in the *Confessions*, wherein the basic model is musical perception. He recognized an eternal measure that should imply beauty and salvation, dependent on whether one is able to recognize the inherent *dynamis* (in Greek, δύναμις – potentiation, power, ability) constituting substantial geometry.²²

Music metric and the perception of time appear to be so profoundly interconnected that their definitions have often ended being circular. By means of a clepsydra, Herophilus measured the durations of heart contraction and dilatation and related these units to metrical music rhythms.²³ Pouring over a page of Galileo's notebooks, Stillman Drake conjectured that in his experiments on free fall, Galileo measured time by counting music bars played at a particular tempo.²⁴ Stravinsky considered that music is an irreplaceable order, strongly resembling architecture, which regulates a relation to time.²⁵ In the same vein, Goethe regarded architecture to be petrified music. It indicates a vertical temporality that has arisen through the duration of the complex instant which is vertically designed.²⁶

Time in the Plastic Arts

Artistic time has been discussed in an article by Étienne Souriau, which principally refers to the plastic arts (design, painting, sculpture, architecture, etc.) in his claim:

²⁰ Ibid., 39–50.

²¹ Aurelio Agostino, *Ordine, musica, bellezza*, Milano, Rusconi, 1992, 90–93, 246.

²² Aurelio Agostino, *Le confessioni*, Edizione Acrobat, 89.

²³ Claudia Arozqueta, "Heartbeats and the Arts: a Historical Connection", *Leonardo*, 51 (1), 2018, 33–39.

²⁴ Stillman Drake, "The Role of Music in Galileo's Experiments", *Scientific American*, 232(6), 98–104.

²⁵ Massimo Donà, op. cit., 129–143.

²⁶ Jonathan D. Kramer, op. cit., 54–57.

Nothing is more dangerous for the exact and delicate understanding of the plastic arts than their rather banal description as the 'arts of space' in contrast to phonetic and cinematic arts characterized as 'arts of time'. This contrast, subscribed to by a great number of aestheticians from Hegel to Max Dessoir, has its historic origin in the philosophy of Kant, particularly in the contrast he makes between the external senses, to which the form of space would be inherent, and the internal sense whose form would be time. The desire to bring music and poetry into the realm of the internal sense (in order to see there 'the soul speaking directly to the soul') has often led to a real misunderstanding of the extent and the cosmic reach of the plastic arts, stripped of their temporal dimensions, and of their content according to that dimension.²⁷

In order to overcome evident deficiencies, he postulates the concept of intrinsic time defined to be "the artistic time inherent in the texture itself of a picture in its composition, in its aesthetic arrangement". Focusing on the analysis of illustrations, Souriau still fails to realize the definition in full.

Such a view of spatiality implies temporal organization relating to dynamism and creativity, which is termed by Bachelard *poetics of space*, indicating a significant link to vertical temporality. He highlights that the poetic image is not subject to causal necessity, which is corroborated by the originality of the image as well as by the influence on observers that have not been involved in its creation. One finds therefore that communicability is a fact of great importance, which makes the concept of causality to be transfigured into information.²⁸ In that regard the statistical causality is required, whose informational content corresponds to the original creation in art.²⁹

Bachelard *inter alia* suggests a substantial geometry of the poetic image about a house that is expanding towards infinity and becoming an immense cosmic house. It allows the poet to inhabit the Universe which is coming to settle the house.³⁰ Except poetry and verses quoted by Bachelard, the image is amazingly emergent in the Orthodox iconography wherein it has become a representation of spatiality. The authors refer to the traditional iconography of the Byzantine style that occurred after the iconoclastic period in the Eastern Roman Empire. Becoming a substantial constituent of the Orthodox rite,

²⁷ Étienne Souriau, op. cit.

²⁸ Gaston Bachelard, 1961, op. cit., 2.

²⁹ Miloš Milovanović, Gordana Medić-Simić, op. cit.

³⁰ Gaston Bachelard, *La poétique de l'espace*, Paris, Les presses univrsitaires de France, 1961, 61.

the icon is regarded as a complex instant of the vertical temporality which is an incarnation of redemptive history.³¹

The icon of the Last Supper in comparison to da Vinci's painting of the same motif indicates a significant deviation (Fig. 1). The Renaissance image narrows toward the interior, placing Christ's figure in the central position where the rays of linear perspective intersect. On the other hand, the icon expands toward the interior with perspective rays that intersect at some elements of the image, but also emerge from the image, implying an observer to which the substantial geometry is subjected (a vacant place by the table). In that manner, time appears to become a depth dimension representing the observer's participation in the icon.³² It is a defining feature of the Orthodox iconography, which was termed *reverse perspective* by the Russian mathematician Pavel Florensky.³³ In contrast, the Renaissance image eliminated time by implementing a photographic realism.

In the icon of Christ's Transfiguration, the reverse perspective is implemented by means of mountain massifs expanding towards the interior (Fig. 2). Time is recognized in a progression from the horizontal through the semi-vertical finally to the vertical position with respect to Christ's central figure.³⁴ The spatial structure is indicated by an emergence of the von Koch curve whose scaling properties concern the dynamism of the image. It corresponds to fractal geometry, wherein time grasped through scaling has arisen in accordance to the manner in which the figures are constructed. Fractals are chimerical forms that imply "time inherent in the texture itself of a picture",³⁵ which is a striking cognation to iconography. In that respect, the spatial structure entails temporal organization which is regarded to be a fundamental principle of arts.³⁶

³¹ Konstaninos D. Kalokyris, "Byzantine Iconography and 'Liturgical' Time", *Eastern Churches Review*, 1(4), 1967, 142–149.

³² Miloš Milovanović, Bojan M. Tomić, op. cit.

³³ Pavel Florensky, "Obartanaya perspektiva", in: *Sochineniya v 4-h tomah*, vol. 3, part 1, Moskva, Myslie, 1999, 46–58.

³⁴ Miloš Milovanović, Bojan M. Tomić, op. cit.

³⁵ Étienne Souriau, op. cit.

³⁶ Clemena Antonova, op. cit., 5.

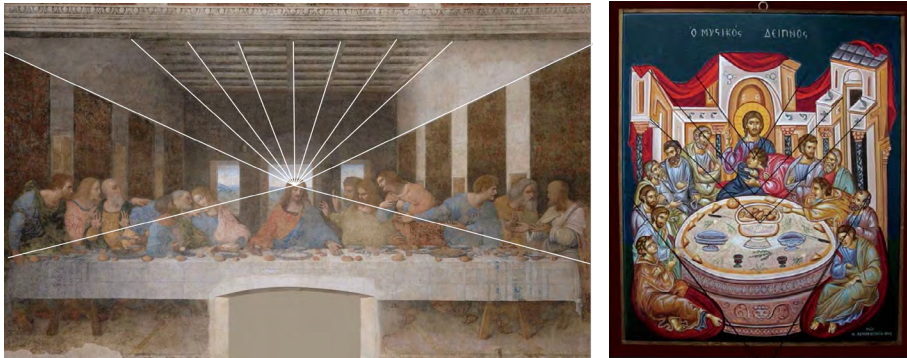


Figure 1. *The Last Supper*

Left: The Renaissance painting. Christ occurs in the center where the rays of linear perspective intersect. Right: The Orthodox icon. The observer occurs in the center where the rays of reverse perspective intersect.



Figure 2. *Christ's Transfiguration*

Left: The expansion of the mountain massifs implements the reverse perspective which is recognized in the progression from the horizontal through the semi-vertical, finally to the vertical position in respect to the central figure of Christ. Right: Emergence of the Von Koch curve elucidates the scaling properties of the icon.

Results

The Time Operator Formalism

According to Jonathan Kramer, the special time sense evoked by music recalls its origins concerning ritual. It relates time to the sacred origin, implying a dual nature of being and becoming which are both fundamental for

artistic articulation.³⁷ The claim applies to Orthodox iconography as well, due to the concept of *liturgical time* which indicates a significant link to the conception presented by Souriau.³⁸ The spatial structure of artworks implies, therefore, a temporal organization which is regarded to be a substantial component of arts in general.

The dual nature of time fits into the physics of complex systems elaborated by Ilya Prigogine whose engagement predominantly concerns the unification of reversible and irreversible theories.³⁹ Referring to the Koopman-von Neumann formalism of statistical mechanics,⁴⁰ he considers the evolution of random variables by a one-parameter group of unitary operators U^t whose adjoints $U^{t\dagger}$ should evolve probability distributions over the phase space. A defining feature of complex systems is the intrinsic time operator T that has satisfied the uncertainty principle in terms of the commutator relation $[T, U^t] = tU^t$ which comes down to $[T, U] = U$ for the group generator U .

The intrinsic time corresponds to a creative behavior and, in that regard, Prigogine suggests the term *self-organization* which means the emergence of structure and innovation in a system.⁴¹ It implies statistical causality containing information termed *statistical complexity* which is the measure of such behavior.⁴² A change in representation $\Lambda = \lambda(T)$ which is the operator function of time maps with no information loss the reversible evolution to an irreversible one, of which the first is represented by the group U^t and the second one by the semigroup

$$W^t = \Lambda U^t \Lambda^{-1}, t \geq 0$$

Although there is also an inverse operator $W^{-t} = \Lambda U^{-t} \Lambda^{-1}$, the adjoint $W^{-t\dagger}$ is not positivity preserving and therefore not corresponding to an evolution that maps one probability distribution to another.⁴³

³⁷ Jonathan D. Kramer, op. cit., 16–19.

³⁸ Konstaninos D. Kalokyris, op. cit.

³⁹ Ilya Prigogine, *From Being to Becoming: Time and Complexity in Physical Science*, New York, W. H. Freeman & Co., 1980.

⁴⁰ Bernard O. Koopman, “Systems and Transformations in Hilbert Space”, *Proceedings of the National Academy of Sciences*, 17(5), 1931, 315–318.

⁴¹ Ioannis Antoniou, Baidyanath Misra, Zdzislaw Suchanecki, *Time Operator, Innovation and Complexity*, New York, John Wiley & Sons, 2003.

⁴² James P. Crutchfield, “What Lies between Order and Chaos”, in: John Casti, Andrew Karloqvist (Eds), *Art and Complexity*, Amsterdam, Elsevier, 2003, 31–45.

⁴³ Baidyanath Misra, Ilya Prigogine, Maurice Courbage, “From Deterministic Dynamics to Probabilistic Descriptions”, *Physica A: Statistical Mechanics and Its Applications*, 98. (1–2), 1979, 1–26.

The spatial organization of complex systems relates to fractal geometry, which is the term coined by Mandelbrot from a Latin adjective *fractus* meaning fractured but also irregular in the manner of fragmentation.⁴⁴ The examination of fractals requires a subtle form of the recurrent order that is termed *self-similarity*, which is a hierarchical design figuring the pattern at successive scales. It is conceived to be a generative property related to the growth of an organism, in each particular case defining the specific geometry of the matter discussed (Fig. 3). Though it involves biological systems, as well as social and cosmic ones, it is manifested in a paradigmatic form by a tree branching, which indicates the intrinsic time of fractal geometry.⁴⁵



Figure 3. Branching of the tree, which indicates the intrinsic time. Left: *The Tree of Life* which is a fractal form in iconography. Right: Self-similarity which is a hierarchical design figuring the fractal at successive scales.

Self-similarity concerns the vertical design of the signal space, which is related to wavelet bases constructed by translations and dilatations of a single function termed the *mother wavelet*. The time operator in regard to the base has eigenvalues corresponding to the scale of a basic eigenfunction.⁴⁶ The

⁴⁴ Benoît Mandelbrot, *The Fractal Geometry of Nature*, San Francisco, W. H. Freeman & Co., 1982.

⁴⁵ Miloš Milovanović, Gordana Medić-Simić, op. cit.

⁴⁶ I. E. Antoniou, K. E. Gustafson, “The Time Operator of Wavelets”, *Chaos, Solitons and Fractals*, 2000, 11(1–3), 443–452.

decomposition in the wavelet base enables establishing a statistical model of the signal space, termed the *hidden Markov model* whose states represent causality in a system. Information contained in the causal states is statistical complexity that indicates the self-organization of the system discussed.⁴⁷

The Complex System Model

In order to establish a signal processing model, one implies the space $L^2(I)$ consisting of one-dimensional signals of a finite energy over the timeline $I=[0,1]$. A signal $f \in L^2(I)$ is decomposed in terms of a wavelet base

$f = A + \sum_{j=0}^{\infty} \sum_{k=1}^{2^j} D_{j,k} \psi_{j,k}$ whereat j corresponds to the dyadic scale and k to the position of a basic element $\psi_{j,k}$. A designates an average value that is a projection onto the subspace of constants and $D_{j,k}$ are detail coefficients, each being inherited by two of them at the next scale that share its support since the scale succession halves a wavelength of the basic element.⁴⁸ Due to the inheritance, detail coefficients are supplied by a hierarchy of the binary tree (Fig. 4).

Elaborating the statistical model of the tree $D = (D_{j,k})$, detail coefficients are regarded to be random variables. In that respect, one considers the extended space $L^2(I \times I)$ containing variables $F: I \rightarrow L^2(I)$ whose codomain is constituted of signals $f \in L^2(I)$. The correlation in a signal mostly concerns inheritance along branches of the binary tree, occurring only through the hidden variables $S = (S_{j,k})$. The wavelet-domain hidden Markov model, which is founded in that manner, has been proven tremendously useful in a variety of applications including speech recognition and artificial intelligence.⁴⁹

The hidden variable $S_{j,k}$ at a particular node might take two state values, designated by α and ω , whereby the first one corresponds to a high variance conditional distribution $D_{j,k} | \alpha$ of the detail coefficient at a node and the second one to a low variance conditional distribution $D_{j,k} | \omega$. In that respect, one

⁴⁷ Miloš Milovanović, Milan Rajković, “Quantifying Self-organization with Optimal Wavelets”, *Europhysics Letters*, 2013, 102(4), 40004.

⁴⁸ Ingrid Daubechies, *Ten Lectures on Wavelets*, Philadelphia, Society for Industrial and Applied Mathematics, 1992, 304–307.

⁴⁹ Matthew S. Crouse, Robert D. Nowak, Richard G. Baraniuk, “Wavelet-based Statistical Signal Processing Using Hidden Markov Model”, *IEEE Transactions on Signal Processing*, 46(4), 1998, 886–902.

of them is related to random and the other to the ordered component of complex behavior. Considering the statistical complexity to be an amalgam of order and randomness,⁵⁰ information stored in the hidden variable corresponds to the complex behavior of the detail coefficient. Respecting that, the Markovian tree $S = (S_{j,k})$ identifies statistical causality containing information $H(S)$ that is the measure of the system's self-organization in regard to the wavelet base.⁵¹ Authors imply the Shannon information $H(\cdot) = -\sum p_n \log p_n$ wherein p_n 's designate probabilities a random variable to take diverse values.

In order to elucidate an evolution of the model, one considers symmetry of the binary tree related to the scaling of a signal. The detail coefficients $D'_{j+1,k}$ of the signal Uf obtained by a scale shift U should correspond to coefficients $D_{j,k}$ of the signal f . Solving the equation $D'_{j+1,k} = D_{j,k}$, one obtains the operator $Uf(\omega) = \begin{cases} f(2\omega) & 0 \leq \omega < \frac{1}{2} \\ f(2\omega-1) & \frac{1}{2} < \omega \leq 1 \end{cases}$ induced by an expansion of the timeline via

Rényi's map $R(\omega) = \begin{cases} 2\omega & 0 \leq \omega < \frac{1}{2} \\ 2\omega-1 & \frac{1}{2} < \omega \leq 1 \end{cases}$ which is a measure preserving the trans-

formation of the unit interval. The evolutionary operator U is non-invertible, but it extends to an invertible one depending on the wavelet base. The extended operator acts on the tensor product $L^2(I \times I) = L^2(I) \otimes L^2(I)$ which the signal space $L^2(I)$ is embedded in.⁵²

The intrinsic time in regard to a wavelet base $T\psi_{j,k} = j \cdot \psi_{j,k}$ takes eigenvalues corresponding to the scale of a basic eigenfunction.⁵³ Considering the evolution of a wavelet base, one obtains $[T, U]\psi_{j,k} = TU\psi_{j,k} - UT\psi_{j,k} = U\psi_{j,k}$ whence it follows $[T, U] = U$ which is the commutator relation that has defined the time operator. The relation holds as well for an evolutionary group which is generated by the extension of U , providing that T has been extended in the same manner.

In terms of the time operator formalism, there is a change in representation mapping the group to a semigroup with no loss of information. The irreversible evolution by the semigroup corresponds to blurring of the signal, due to the mechanism of cumulative listening which concerns the expansion

⁵⁰ James P. Crutchfield, op. cit.

⁵¹ Miloš Milovanović, Milan Rajković, op. cit.

⁵² Miloš Milovanović, Srđan Vukmirović, Nicoletta Saulig, "Stochastic Analysis of the Time Continuum", *Mathematics*, 9(12), 2021, 1452.

⁵³ I. E. Antoniou, K. E. Gustafson, op. cit.

of the timeline via the action of the evolutionary operator onto the signal space.⁵⁴

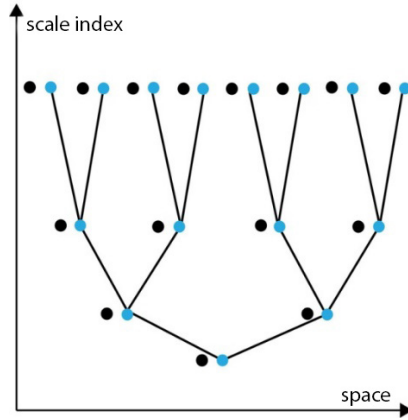


Figure 4. The wavelet-domain hidden Markov model. The black colour denotes detail coefficients and the blue, hidden variables.

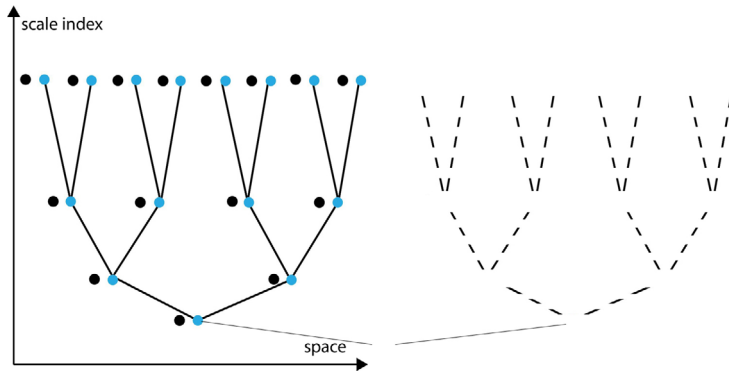


Figure 5. The mechanism of cumulative listening. The expansion of the timeline in order to involve one more scale, stepping backward into the intrinsic time.

⁵⁴ Jonathan D. Kramer, *op. cit.*, 367–370.

Discussion

Aesthetics of Verticality

Kramer first used the term *statistical* and then *stochastic* in order to refer to cumulative listening.⁵⁵ The word *listening* should be considered metaphoric, but one really does accumulate information on a signal which is a mechanism that implies stepping backward with regard to the intrinsic time (Fig. 5). The horizontal timeline is therefore opposed to the intrinsic time that is vertical in regard to that. Kramer considers vertical time to be “a temporal continuum of the unchanging, in which there are no separate events and everything seems to be part of an eternal present”.⁵⁶ He indicates that a deep listening allows to transcend the time music takes and experience the time it evokes. The horizontal axis corresponds to the spatial frame in the same manner as it exists in the plastic arts. On the other hand, the vertical one concerns the intrinsic time being irreducible to spatiality.

Discussing the aesthetics of verticality, Kramer refers to John Cage who believed that “music fulfills itself when it teaches people how to listen, so that they may end up preferring the trivial noises of daily life to music”.⁵⁷ What it lacks, however, in noise for musical consideration is the absence of complex behavior. The complexity of noise is low in any base, which impedes from being art. Cage’s position is actually a utopian solution of discrepancy between art and reality. However, the claim that his music is not “an attempt to bring order out of chaos nor to suggest improvements in creation, but simply a way of walking up to the very life we are living” indicates that the reality is not figured in terms of self-organization. Cage and other composers working in the mid-century attached great importance to randomness and spoke often of its use in composition. But one almost needs to mistrust their own statements about what they were doing. There was really an interest in exploring complex systems, whose terminology was not available to the artists. The concept of American minimalism is to pare down the systems involved to the point that complexity arises out of the situation.⁵⁸

⁵⁵ Ibid., 408–409.

⁵⁶ Ibid., 454.

⁵⁷ Ibid., 384–386; Wim Mertens, *American Minimal Music*, New York, Alexander Broude, 1983, 109.

⁵⁸ Tim Parkis, “Complexity and Emergence in the American Experimental Music Tradition”, in: John Casti, Andrew Karlovist (Eds), *Art and Complexity*, Amsterdam, Elsevier, 2003, 75–84.

Kramer remarks that vertical music can be totally non-linear, or otherwise so totally linear that predictability reigns.⁵⁹ The reason for such a paradoxical view is that he does not regard the statistical complexity of a system. The absence of complex behavior does not constitute any temporality, but it is reduced to an extreme, whether deterministic or random. The trouble occurs because of a dualistic conception, relating vertical time to the right brain consciousness.⁶⁰ On the contrary, verticality should be an integral consciousness relieved from the dual extremes of left and right brains.⁶¹ It is related to the time operator formalism, implying statistical causality whose informational content corresponds to the complexity measure. Vertical music is therefore an extremely complex one and not the music deprived of any structure. Kramer correctly observes that some pieces involve considerable structuring of the compositional process.⁶² However, causality occurs through inheritance in the vertical time. Kramer's failure concerns the usage of redundancy to quantify verticality, which is actually equivalent to the entropy rate.⁶³ But it is not an adequate measure for such a quantification, requiring the statistical complexity that is complementary to entropy and redundancy.⁶⁴ Furthermore, he relates linearity to directionality which is one more misconception. Regarded to be a spatial frame, the timeline is actually non-directional. The direction appears due to self-organization that is represented by vertical temporality. Respecting that, music offers a myriad of temporal experiences which are classified by Kramer into a multitude of categories.⁶⁵

An intrinsic time designing innovation and creativity, that is opposed to the timeline expansion, has been indicated by Matthew MacDonald, who considers the music of Charles Ives in terms of breaking time's arrow.⁶⁶ Ives' music seems to lack the timeline succession and coherence relative to classical tradition, which is due to an extreme fragmentation that has often characterized musical surfaces resembling fractal geometry. In an essay by Robert Morgan, the fragmentation is identified to be one of the spatial features in Ives' music designed "to negate, as much as possible, the succession of tem-

⁵⁹ Ibid., 61.

⁶⁰ Ibid., 9–12.

⁶¹ James P. Crutchfield, op. cit.

⁶² Jonathan D. Kramer, op. cit., 56.

⁶³ Ibid., 389.

⁶⁴ James P. Crutchfield, op. cit.

⁶⁵ Jonathan D. Kramer, op. cit., 9.

⁶⁶ Matthew MacDonald, *Breaking Time's Arrow: Experiment and Expression in the Music of Charles Ives*, Bloomington, Indiana University Press, 2014.

poral sequence as the principal path for establishing a musical relationship”.⁶⁷ He noted that in the song *The Things Our Fathers Loved* “it is almost as if the whole cause-and-effect pattern of traditional tonal music has been turned upside down”. But characterizing Ives’ music as a spatial one, Morgan overplayed his denial of musical temporality. The temporal organization and cause-and-effect relationship are substantial, even in an extremely fractured environment.⁶⁸ In that regard, one requires a complex system model providing statistical causality which is related to inheritance within the vertical time that should refer to Bachelard’s conception. The information it contains corresponds to self organization representing the originality issue.⁶⁹

A listener is encouraged to experience the *a priori* succession by transfiguring fragments through an aural equivalent of assembling the jigsaw puzzle.⁷⁰ The process of aurally reassembling music corresponds to fragmented manuscripts of Ives and associated compositional habits, which is considered in detail by MacDonald. The vertical temporality corresponds to his own imagination, applying to perception and creation of art as well. A further elaboration of the topic should present a comprehensive view concerning the creativity of the composer.

According to MacDonald, “Ives’ music registers the desire for temporal transcendence, drawing on a music of the past to create the music of the future. In this way, Ives’ view of the past and the future are complementary, not paradoxical.”⁷¹ The conception of time could undo the pastness of the past, moving it from the realm of what-has-been into the realm of what-will-be, and ultimately simply what-is. Music, so conceived, strives to master time bringing it to a halt. It is aimed at capturing an intrinsic time, holding onto it in the present, which corresponds to Bergson’s concept of duration.⁷² The conception by Henri Bergson has been incorporated in the time operator formalism of complex systems, constituting the fundament of the model which should unite the concepts by Bachelard, Souriau, Kramer and MacDonald as well.⁷³

⁶⁷ Robert P. Morgan, “Spatial Form in Ives”, in: H. Wiley Hitchcock, Vivian Perlis (Eds), *An Ives Celebration: Papers and Panels of the Charles Ives Centennial Festival-Conference*, Urbana, University of Illinois Press, 1996, 145–158.

⁶⁸ MacDonald, op. cit., 7–10.

⁶⁹ Miloš Milovanović, Gordana Medić-Simić, op. cit.

⁷⁰ MacDonald, op. cit., 10.

⁷¹ Ibid., 7–10.

⁷² Ibid., 137–138.

⁷³ Ioannis Antoniou, Theodoros Christidis, “Bergson’s Time and the Time Operator”, *Mind and Matter*, 8(2), 2010, 185–202.

Art of Memory

Criticizing the photographic medium and its claims in regard to fidelity and truthfulness, Siegfried Kracauer asserts the potential of film to organize photographs according to a contextual significance they lack. The model of valid organization he implies is memory, which should concern arrangement aligned to a peculiar perspective that the individual instances do not capture. Kracauer refers to Dupont's remark that "the essence of film is, to a certain extent, the essence of time", indicating a fundamental consciousness which is figured in terms of cognitive structures. It is characterized by a deep sense and emphasized emotions, causing a chronological measure that is not directly affected by linear time.⁷⁴

Tarkovsky, Pasolini and others have considered the transfiguration of the timeline by structuring a vertical temporality to be the main problem of film.⁷⁵ Film testifies to an aesthetic principle whose substance is not mere illustration, but it is about imprinting time on celluloid which is termed by Andrei Tarkovsky as *sculpturing in time*. In addition to the fact that film theorists speak of it as a composite art that unites drama, prose, acting, painting, architecture, music, etc., he underlines that film is not an aesthetic amalgam of other principles and methods of expression. Film itself is time, and the basic ingredient permeating it concerns temporality.⁷⁶ In that regard, Arnold Hauser states that "through an analysis of time, film enabled the visual expression of an experience which could have been expressed only in music before".⁷⁷

As Walter Pater famously observed "all art constantly aspires towards the condition of music",⁷⁸ which implies an architecture of the complex instant constituting an extra dimension that spreads into depth and height.⁷⁹ Gravitation is considered in its possibility rather than in terms of a necessary development. Vertical depth gives the Earth center a significant role, which is a

⁷⁴ Siegfried Kracauer, "Photography", *Critical Inquiry*, 193, 1993, 421–436.

⁷⁵ Nono Dragović, *Poetika filmske režije: kinetička teorija filma* (The Poetics of Film Direction), Pančevo, Mali Nemo, 2008, 76–78.

⁷⁶ Andrei Tarkovsky, *Sculpting in Time: Tarkovsky The Great Russian Filmmaker Discusses His Art*, Austin, University of Texas Press, 1989, 62–67.

⁷⁷ Arnold Hauser, *Socijalna istorija umetnosti i književnosti* (The Social History of Art), vol. 2, Beograd, Kultura, 1966, 448.

⁷⁸ Charles Jencks, "Architecture Becomes Music", *Architectural Review*, 233 (1395), 2013, 91–108.

⁷⁹ Herbert A. Simon, "The Architecture of Complexity", *Proceedings of the American Philosophical Society*, 106(6), 1962, 467–482.

pretty good idea of how causal necessity has been flashed out.⁸⁰ It is organized and completed in order to be an emergent phenomenon.⁸¹ There is a substantial relation to neural architecture whose structure reflects the depth concerning cognitive complexity.⁸² Respecting that, complex patterns inevitably involve a temporal aspect whose recognition is incident to predictability of the behavior.⁸³

The scientific aesthetics by Milutin Borisavljević considers architecture to be the art of time.⁸⁴ He obtained a doctoral degree in aesthetics under the mentorship of Victor Basch whose student was Souriau as well.⁸⁵ Borisavljević discerns an element of motion in architectural design, that communicates through rhythms, realizing emotional states which result in creativity and innovation. Analogously to the mechanism of cumulative listening, he perceives the existence of consecutive images that relate architecture to film and music. It is the memory process that creates an image of the event and gradually accumulates encoded information in the form of a mental representation, since large-scale proportions are perceived only in respect to the cumulative listening.⁸⁶

The art of memory has been employed not only for retaining music which was already composed, but also to induce artistic creativity due to ability to manipulate the matter.⁸⁷ Almost every mnemotechnical scheme (ladders, roses, buildings, maps) is based on geometrical figures (squares, rectangles, triangles, circles) and complex reformations of those, including three dimensional structures.⁸⁸ Frances Yates indicated that the usage of architectural features in Freemasonry should refer to its origins concerning the art of

⁸⁰ Gaston Bachelard, 2016, op. cit.

⁸¹ Erik P. Verlinde, "Emergent Gravity and the Dark Universe", *SciPost Physics*, 2(3), 2017, 016.

⁸² Thomas Sambrook, Andrew Whiten, "On the Nature of Complexity in Cognitive and Behavioural Science", *Theory & Psychology*, 7(2), 1997, 191–213.

⁸³ Miloš Milovanović, Gordana Medić-Simić, op. cit.

⁸⁴ MiloutineBorissavliévitch, "L'architecture art du temps", *Construction moderne*, 34, 1925, 404–408.

⁸⁵ Irena Kuletin Čulafić, *Naučna estetika arhitekture Milutina Borisavljevića* (Miloutine Borissavliévitch and His Scientific Aesthetics of Architecture), Beograd, Arhitektonski fakultet, 2012, 179–190.

⁸⁶ Jonathan D. Kramer, op. cit., 367–370.

⁸⁷ Ana Maria Busse Berger, *Medieval Music and the Art of Memory*, University of California Press, Berkeley, 2005, 7.

⁸⁸ Mary Carruthers, Jan M. Ziolkowski (Eds), *The Medieval Craft of Memory: An Anthology of Texts and Pictures*, University of Pennsylvania Press, 2002, 16.

memory.⁸⁹ The complex Ark of Noah, that was constructed by Hugh of Saint-Victor, represents a usage of the architectural design in memorization.⁹⁰

The wavelet domain hidden Markov model derived for one-dimensional signals is easily generalizable to two-dimensional or three-dimensional ones using 2D or 3D wavelet transform. The model provides a neuroaesthetical computation in order to recognize the statistical causality of the signal.⁹¹ It resembles the predictive coding model in the cognitive neuroscience where every level of neural hierarchy makes predictions and propagates to lower levels (top-down) being compared to a stimulation, whilst predictions from lower levels propagate in the opposite direction (bottom-up). The comparison generates a prediction discrepancy and repairation until it becomes complete.⁹² A substantial geometry the model implies is rendered by fractals that are chimerical forms entailing a temporal sensation. In that respect, the contemporary architects Zaha Hadid, Frank Gehry and others have utilized fractal geometry that is far more receptive for an aesthetic sense of art perception.⁹³ Architectural structures which are provided by vertical temporality give rise to the postmodern architecture rendered by fractal forms.⁹⁴

Fractal design corresponds to the tree structure which is the view on imagination by Bachelard.⁹⁵ The complex system model elucidating the poetic image has required statistical causality that occurs due to inheritance in the vertical temporality. It is in the vertical time of complex instant that poetry finds a specific dynamism. For those who know how to read Poe's *The Raven*, midnight no longer flows by the timeline, but striking "deeper and deeper within the soul".⁹⁶ The image of a "tall tree within the ear", in Rilke's *Sonnets to Orpheus* who is the most primordial of all musicians, indicates a quest for the depth that has resulted by a "temple deep inside their hearing".⁹⁷

⁸⁹ Frances Yates, *The Art of Memory*, London, Ark Paperback, 1984.

⁹⁰ Mary Carruthers, Jan M. Ziolkowski (Eds), op. cit., 41–70.

⁹¹ Miloš Milovanović, Gordana Medić-Simić, op. cit.

⁹² James M. Kilner, Karl J. Friston, Chris D. Frith. "Predictive Coding: an Account of the Mirror Neuron System", *Cognitive Processing*, 8, 2007, 159–166.

⁹³ Michael J. Ostwald, Josephine Vaughan, *The Fractal Dimension of Architecture*, Basel, Birkhäuser, Springer International Publishing Switzerland, 2016.

⁹⁴ Charles Jencks, *The New Paradigm in Architecture: The Language of Postmodernism*, New Heaven and London, Yale University Press, 2002.

⁹⁵ Gaston Bachelard, 1971, op. cit.

⁹⁶ Gaston Bachelard, 2013, op. cit., 59–61.

⁹⁷ Rainer Maria Rilke, "The Sonnets to Orpheus (No.1)", in: *The Selected Poetry of Rainer Maria Rilke*, New York, Vintage, 1989, 227.

In that manner, the poetic imagination is related to the sacred origins of music which refer to verticality.⁹⁸

A whisper of trees which rustle in poem *The Infinity* by Giacomo Leopardi is straightforwardly linked to the sensation of vertical time as well.⁹⁹ The power spectrum of the white noise model for tree whispering is a constant, whence it follows the representation in the lag domain by the delta function that is a singular distribution whose support is pointwise. The irreversibility requires a change in representation that should delocalize the conceptual structure describing it by elementary entities which are not mere points.¹⁰⁰ The basic object is no more the point and its evolution along a trajectory by the action of U^t , but the transformation Λ of points evolving by W^t which is an intensional view.¹⁰¹ The transformation Λ has involved a context of the observation, which is missing in the extensional view consisting of single points only.¹⁰² An inverse transformation from irreversible to reversible dynamics is impossible to realize, since it eliminates contextual dependence which is concerned with the vertical time. The peculiar verb tense opening Leopardi's poem points on an eternal recurrence of the complex instant, which is a temporality that ties up all points from the horizontal timeline. It is an instance of the iconographic vision which is marked by the rejection of the Renaissance perspective that is performed from a single angle.¹⁰³

Bachelard declared that he had accepted everything from Bergson, except continuity in terms of the horizontal timeline.¹⁰⁴ The uprise of the relativity theory brought to ruin all arguments that had relied on a unique duration which should be a fundamental principle for ordering events.¹⁰⁵ The well-specified instant however remains an absolute *chronotope* (in Greek,

⁹⁸ Jonathan D. Kramer, op. cit., 16–19.

⁹⁹ Nicoletta Saulig, op. cit.

¹⁰⁰ Baidyanath Misra, Ilya Prigogine, "Irreversibility and Nonlocality", *Letters in Mathematical Physics*, 7, 1983, 421–429.

¹⁰¹ Miloš Milovanović, Nicoletta Saulig, "An Intensional Probability Theory: Investigating the Link between Classical and Quantum Probabilities", *Mathematics*, 2022, 10, 4294.

¹⁰² Servet Martinez, Enrique Tirapegui, "A Possible Physical Interpretation of the Λ Operator in the Prigogine Theory of Irreversibility", *Physics Letters A*, 110(2), 1985, 81–83.

¹⁰³ Davide Messina, "Blind Windows: Leopardi with Rothko", *Nineteenth-Century Contexts*, 41(1), 2019, 51–62.

¹⁰⁴ Jean François Peraudin, "Bachelard's 'Non-Bergsonism'", in: Eileen Rizo-Patron (Ed.), *Adventures in Phenomenology: Gaston Bachelard*; New York, Sunny Press, 2017, 19–47.

¹⁰⁵ Jimena Canales, *The Physicist and the Philosopher: Einstein, Bergson, and the Debate That Changed Our Understanding of Time*, Princeton University Press, 2015.

χρόνος – time τόπος – space), constituted of both horizontal duration and time which is vertical in regard to that.¹⁰⁶ Bachelard's hermeneutic shift was initially inspired by the scientific revolution in physics including the uncertainty principle and theory of relativity,¹⁰⁷ which is considered to be a prelude of postmodernism in science.¹⁰⁸ Given his evident attraction to alchemical attainment, one should recognize an aesthetical encounter of premodern and postmodern science in the physics of complex systems upon which the model is based.¹⁰⁹ Alchemy perceives matter in terms of becoming, which makes time a substantial component.¹¹⁰

The conception by Bachelard is extremely similar to the liturgical time of Orthodox iconography, but instead of that he commonly refers to poetic and classical allusions. The poetics of space is however a vertical one, resembling Jacob's ladder which is a scale connecting Earth to Heaven.¹¹¹ The concept of chiasm by Maurice Merleau-Ponty also refers to a crossing between the visible and the invisible, exhibiting reversibility that is immanent but never fully realized.¹¹² Such intertwining makes art to be a language in its own right, since it generates meaning through the exchange of sensation and expression.¹¹³ Integrating both reversible and irreversible time in an artistic mythology has actually transcended the gap between being and becoming.¹¹⁴ There is no distinct crossing anymore, but the visible and the invisible are united by means of a depth which is designed by the intrinsic time operator.

¹⁰⁶ Bachelard 2013, op. cit., 17.

¹⁰⁷ Richard Kearney, op. cit.

¹⁰⁸ Stephen Toulmin, *The Return to Cosmology: Postmodern Science and the Theology of Nature*, Berkeley, University of California Press, 1985.

¹⁰⁹ Miloš Milovanović, Gordana Medić-Simić, op. cit.

¹¹⁰ Paulina Gurgul, "Alchemy of Words: Gaston Bachelard's Theory of Imagination", in: Nataša Janković, Boško Drobnjak, Marko Nikolić (Eds), *Proceedings of the 21st International Congress of Aesthetics*, Belgrade, Faculty of Architecture, 22–26 August 2019, 418–423.

¹¹¹ Richard Kearney, op. cit.

¹¹² Maurice Merleau-Ponty, *The Visible and the Invisible*, Evanston, Northwestern University Press, 1968, 130–155.

¹¹³ Jessica Wiskus, "Inhabited Time: Couperin's *Passacaille*", in: Ana-Teresa Tymieniecka (Ed.), *Logos of Phenomenology and Phenomenology of the Logos. Book Three: Logos of History-Logos of Life. Historicity, Time, Nature, Communication, Consciousness, Alterity, Culture*, Dordrecht, Springer Netherlands, 2006, 177–193.

¹¹⁴ Claude Lévi-Strauss, op. cit.

Conclusion

The paper is intended to elucidate the concept of vertical time that originates from Gaston Bachelard. It is extremely similar to the liturgical time of Orthodox iconography, but instead of that he commonly refers to poetic and classical allusions. That is a reason for considering the mythological context of music, due to the existence of a default timeline whose expansion concerns temporal sensation.

The topic is related to the intrinsic time by Étienne Souriau, in considering the plastic arts. The conception fits the time operator formalism of complex systems whose defining feature is the existence of the intrinsic time operator. The complex system model is based upon statistical causality which occurs due to vertical inheritance. The model is derived for one-dimensional signals, but it is easily generalizable to two-dimensional and three-dimensional ones.

The aesthetics of verticality is demonstrated to be significant for music, poetry, architecture and film as well. Referring to various arts, authors have presented a relationship between fractal geometry and statistical causality which provides the model in order to unite diverse concepts by Bachelard, Souriau, Kramer and MacDonald.

Acknowledgements

The authors acknowledge the support of the Ministry of Science, Technological Development and Innovation of the Republic of Serbia through the Mathematical Institute of the Serbian Academy of Sciences and Arts and the University of Belgrade – the Faculty of Architecture, and the Ministry of Science and Education of the Republic of Croatia.

Cited Works

- Agostino, Aurelio: *Ordine, musica, bellezza*. Milano: Rusconi, 1992.
- Agostino, Aurelio: *Le confessioni*. Edizione Acrobat.
- Antoniou, I. E.; K. E. Gustafson: "The Time Operator of Wavelets", *Chaos, Solitons and Fractals*, 2000, 11(1–3), 443–452.
- Antoniou, Ioannis; Baidyanath Misra; Zdzislaw Suchanecki: *Time Operator, Innovation and Complexity*. New York: John Wiley & Sons, 2003.
- Antoniou, Ioannis; Theodoros Christidis: "Bergson's Time and the Time Operator", *Mind and Matter*, 8(2), 2010, 185–202.

- Antonova, Clemena: *Space, Time and Presence in the Icon*. Farnham: Ashgate Publishing Ltd., 2010.
- Arozqueta, Claudia: "Heartbeats and the Arts: a Historical Connection", *Leonardo*, 51(1), 2018, 33–39.
- Bachelard, Gaston: *La poétique de l'espace*. Paris: Les presses universitaires de France, 1961.
- Bachelard, Gaston: *On Poetic Imagination and Reverie*. Putman: Spring Publications, 1971.
- Bachelard, Gaston: *Intuition of the Instant*. Evanston: Northwestern University Press, 2013.
- Bachelard, Gaston: *The Dialectic of Duration*. London: Rowmann & Littlefield International, 2016.
- Beethoven, Ludwig van: *Autobiografia di un genio – Lettere, pensieri, diari*. Milano: Mondadori, 2005.
- Berger, Busse; Ana Maria: *Medieval Music and the Art of Memory*. Berkeley: University of California Press, 2005.
- Borissavliévitch, Miloutine: "L'architecture art du temps", *Construction moderne*, 34, 1925, 404–408.
- Canales, Jimena: *The Physicist and the Philosopher: Einstein, Bergson, and the Debate That Changed Our Understanding of Time*. Princeton: Princeton University Press, 2015.
- Carruthers, Mary; Jan M. Ziolkowski (Eds): *The Medieval Craft of Memory: An Anthology of Texts and Pictures*. Philadelphia: University of Pennsylvania Press, 2002.
- Crutchfield, James P.: "What Lies between Order and Chaos", in: John Casti, Andrew Karloqvist (Eds), *Art and Complexity*. Amsterdam: Elsevier, 2003, 31–45.
- Daubechies, Ingrid: *Ten Lectures on Wavelets*. Philadelphia: Society for Industrial and Applied Mathematics, 1992.
- Donà, Massino: *Filosofia della musica*. Milano: Tascabili Bompiani, 2006.
- Dragović, Nono: *Poetika filmske režije: kinetička teorija filma* (The Poetics of Film Direction). Pančevo: Mali Nemo, 2008.
- Drake, Stillman: "The Role of Music in Galileo's Experiments", *Scientific American*, 232(6), 98–104.
- Florensky, Pavel: "Obartanaya perspektiva", in: *Sochineniyav 4-h tomah*, vol. 3, part 1. Moskva: Myslie, 1999.
- Giovannini, G.: "The Method of Study of Literature in Its Relation to the Other Fine Arts", *Journal of Aesthetics and Art Criticism*, 8(3), 1850, 185–195.
- Gostuški, Dragutin: *Vreme umetnosti: prilog zasnivanju jedne opšte nauke o oblicima*. Beograd: Prosveta, 1968.
- Gurgul, Paulina: "Alchemy of Words: Gaston Bachelard's Theory of Imagination", in: Nataša Janković, Boško Drobnjak, Marko Nikolić (Eds), *Proceedings of the 21st International Congress of Aesthetics. Possible Worlds of Contemporary Aesthetics: Aesthetics Between History, Geography and Media*. Belgrade: Faculty of Architecture, 22–26 August 2019, 418–423.
- Hauser, Arnold: *Socijalna istorija umetnosti i književnosti* (The Social History of Art), vol. 2. Beograd: Kultura, 1966.

- Jencks, Charles: *The New Paradigm in Architecture: The Language of Postmodernism*. New Heaven and London: Yale University Press, 2002.
- Jencks, Charles: "Architecture Becomes Music", *Architectural Review* 233 (1395), 2013, 91–108.
- Kalokyris, Konstaninos D.: "Byzantine Iconography and 'Liturgical' Time", *Eastern Churches Review*, 1(4), 1967, 142–149.
- Kearney, Richard: "Vertical Time: Bachelard's Epiphanic Instant", in: Eileen Rizo-Patron (Ed.), *Adventures in Phenomenology: Gaston Bachelard*. New York: Sunny Press, 2017, 49–61.
- Kilner, James M.; Karl J. Friston; Chris D. Frith: "Predictive Coding: an Account of the Mirror Neuron System", *Cognitive Processing*, 8, 2007, 159–166.
- Koopman, Bernard O.: "Systems and Transformations in Hilbert Space", *Proceedings of the National Academy of Sciences*, 17(5), 1931, 315–318.
- Kracauer, Siegfried: "Photography", *Critical Inquiry*, 193, 1993, 421–436.
- Kramer, Jonathan D.: *The Time of Music*. New York: Schirmer Books, 1988.
- Kuletin Ćulafić, Irena: *Naučna estetika arhitekture Milutina Borisavljevića* (Miloutine Borissavliévitch and His Scientific Aesthetics of Architecture). Beograd: Arhitektonskifakultet, 2012, 179–190.
- Lévi-Strauss, Claude: "The Structural Study of Myth", *Journal of American Folklore*, 68(270), 1955, 428–444.
- MacDonald, Matthew: *Breaking Time's Arrow: Experiment and Expression in the Music of Charles Ives*. Bloomington: Indiana University Press, 2014.
- Mandelbrot, Benoît: *The Fractal Geometry of Nature*. San Francisco: W. H. Freeman & Co., 1982.
- Martinez, Servet; Enrique Tirapegui: "A Possible Physical Interpretation of the Λ Operator in the Prigogine Theory of Irreversibility", *Physics Letters A*, 110(2), 1985, 81–83.
- Mazzola, Guerino; Alex Lubet; Yan Peng; Jordan Goebel; Christopher Rochester; Sangeeta Day: *Making Musical Time*. Springer, 2021.
- Merleau-Ponty, Maurice: *The Visible and the Invisible*. Evanston: Northwestern University Press, 1968, 130–155.
- Mertens, Wim: *American Minimal Music*. New York: Alexander Broude, 1983.
- Messina, Davide: "Blind Windows: Leopardi with Rothko", *Nineteenth-Century Contexts*, 41(1), 2019, 51–62.
- Milovanović, Miloš; Milan Rajković: "Quantifying Self-organization with Optimal Wavelets", *Europhysics Letters*, 2013, 102(4), 40004.
- Milovanović, Miloš; Bojan M. Tomić: "Fractality and Self-organization in the Orthodox Iconography", *Complexity*, 21(S1), 2016, 55–68.
- Milovanović, Miloš; Gordana Medić-Simić: "Aesthetical Criterion in Art and Science", *Neural Computing and Applications*, 2021, 33(6), 2137–2156.
- Milovanović, Miloš; Srđan Vukmirović; Nicoletta Saulig: "Stochastic Analysis of the Time Continuum", *Mathematics*, 9(12), 2021, 1452.

- Milovanović, Miloš; Nicoletta Saulig: “An Intensional Probability Theory: Investigating the Link between Classical and Quantum Probabilities”, *Mathematics*, 2022, 10, 4294.
- Milovanović, Miloš; Nicoletta Saulig: “The Duality of Psychological and Intrinsic Time in Artworks”, *Mathematics*, 12(12), 2024, 1850.
- Misra, Baidyanath; Ilya Prigogine; Maurice Courbage: “From Deterministic Dynamics to Probabilistic Descriptions”, *Physica A: Statistical Mechanics and Its Applications*, 98(1–2), 1979, 1–26.
- Misra, Baidyanath; Ilya Prigogine: “Irreversibility and Nonlocality”, *Letters in Mathematical Physics*, 7, 1983, 421–429.
- Morgan, Robert P.: “Spatial Form in Ives”, in: H. Wiley Hitchcock; Vivian Perlis (Eds), *An Ives Celebration: Papers and Panels of the Charles Ives Centennial Festival-Conference*. Urbana: University of Illinois Press, 1996, 145–158.
- Ostwald, Michael J.; Josephine Vaughan: *The Fractal Dimension of Architecture*. Basel: Birkhäuser, Springer International Publishing Switzerland, 2016.
- Parkis, Tim: “Complexity and Emergence in the American Experimental Music Tradition”, in: John Casti, Andrew Karloqvist (Eds), *Art and Complexity*. Amsterdam: Elsevier, 2003, 75–84.
- Peraudin, Jean François: “Bachelard’s ‘Non-Bergsonism’”, in: Eileen Rizo-Patron (Ed.), *Adventures in Phenomenology: Gaston Bachelard*. New York: Sunny Press, 2017, 19–47.
- Prigogine, Ilya: *From Being to Becoming: Time and Complexity in Physical Science*. New York: W. H. Freeman & Co., 1980.
- Rilke, Rainer Maria: “The Sonnets to Orpheus (No.1)”, in: *The Selected Poetry of Rainer Maria Rilke*. New York: Vintage, 1989.
- Sambrook, Thomas; Andrew Whiten: “On the Nature of Complexity in Cognitive and Behavioural Science”, *Theory & Psychology*, 7(2), 1997, 191–213.
- Saulig, Nicoletta: “Vertical Time: Sound and Vision”, *Kragujevac Journal of Mathematics*, 47(7), 2023, 1065–1074.
- de Saussure, Ferdinand: *Course in General Linguistics*. New York: Philosophical Library, 1959.
- Simon, Herbert A.: “The Architecture of Complexity”, *Proceedings of the American Philosophical Society*, 106(6), 1962, 467–482.
- Souriau, Étienne: “Time in the Plastic Arts”, *Journal of Aesthetics and Art Criticism*, 7(4), 1949, 294–307.
- Tarkovsky, Andrei: *Sculpting in Time: Tarkovsky the Great Russian Filmmaker Discusses His Art*. Austin: University of Texas Press, 1989.
- Toulmin, Stephen: *The Return to Cosmology: Postmodern Science and the Theology of Nature*. Berkeley: University of California Press, 1985.
- Verlinde, Erik P.: “Emergent Gravity and the Dark Universe”, *SciPost Physics*, 2(3), 2017, 016.
- Wiskus, Jessica: “Inhabited Time: Couperin’s *Passacaille*”, in: Ana-Teresa Tymieniecka (Ed.), *Logos of Phenomenology and Phenomenology of the Logos. Book Three: Logos*

of History-Logos of Life. Historicity, Time, Nature, Communication, Consciousness, Alterity, Culture. Dordrecht: Springer Netherlands, 2006, 177–193.

Yates, Frances: *The Art of Memory*. London: Ark Paperback, 1984.

Summary

The paper is intended to elucidate the concept of vertical time that originates from Gaston Bachelard. The conception is extremely similar to liturgical time of the Orthodox iconography, but instead of that he commonly refers to poetic and classical allusions. That is a reason for considering the mythological context of music, due to existence of a default timeline whose expansion concerns temporal sensation. The current article provides a complex system model of the vertical temporality in order to present a universality of the concept corresponding to artistic creativity. In that regard, it should oppose to Kramer's view that verticality concerns particular practices of music only.

The topic is related to the intrinsic time by Étienne Souriau considering the plastic arts. The conception fits to the time operator formalism of complex systems whose defining feature is existence of the intrinsic time operator. The complex system model is based upon statistical causality which occurs due to vertical inheritance. The model is derived for one-dimensional signal, but it is easily generalizable to two-dimensional and three-dimensional ones.

The aesthetics of verticality is demonstrated to be significant for music, poetry, architecture and film as well. Referring to various arts, authors have presented a relationship between fractal geometry and statistical causality which provides the model in order to unite diverse concepts by Bachelard, Souriau, Kramer and MacDonald. The discussion has involved Charles Ives, John Cage, Andrei Tarkovsky, Milutin Borisavljević and other artists which should contribute to in-deep comprehension.